
Biased Random-Key Genetic Algorithms for the Winner Determination Problem in Combinatorial Auctions

Carlos Eduardo de Andrade

andrade@ic.unicamp.br

Institute of Computing, University of Campinas, Avenida Albert Einstein 1251, Campinas, SP 13083-852 Brazil.

Flávio Keidi Miyazawa

fkm@ic.unicamp.br

Institute of Computing, University of Campinas, Avenida Albert Einstein 1251, Campinas, SP 13083-852 Brazil.

Mauricio G. C. Resende

mgcr@research.att.com

Network Evolution Research Department, AT&T Labs Research, 200 S. Laurel Avenue, Middletown, NJ 07748 USA.

Rodrigo Franco Toso

rtoso@cs.rutgers.edu

Department of Computer Science, Rutgers University, 110 Frelinghuysen Road, Piscataway, NJ 08854 USA.

Supplementary material

A Instance tightness

An important aspect of instances for the winner determination problem (WDP) is their tightness. This metric is used by Chu and Beasley (1998) to craft instances of the Multidimensional Knapsack Problem (MDKP), largely used in the literature as the main benchmark for this problem. The tightness of a constraint j is defined as

$$t_j = \frac{c_j}{\sum_{k \in \mathcal{B}} w_{jk}}, \quad (1)$$

where c_j is the availability of resource j and w_{jk} is the amount of resource j requested by k , as defined in Formulation (2) of the main text. Note that for the WDP, the tightness is

$$t_j = \frac{1}{|\{B : j \in B, B \in \mathcal{B}\}|}, \quad \forall j \in M, \quad (2)$$

by definition, i.e., the tightness is defined as the inverse of the number of bids that request a certain good. Note that a low t_j indicates that good j is required by several bids, probably increasing the problem difficulty.

In the Chu and Beasley MDKP instances, every constraint of a given problem has the same tightness, which is either 0.25, 0.5, or 0.75. For the WDP instances, tightness varies for each constraint and depends heavily on the type and size of the problems. For the most classes, as the size increases, tightness decreases, notably for the L2, L7, and LG classes. By definition, for some classes tightness is almost constant as, e.g. L3

and matching. Table 1 shows the average tightness of each constraint for each class and problem size. Note that the hard “path” instances are not shown since the CATS suite does not generate hard instances for “path” distributions.

Table 1: Average of instances tightness.

Class	Size							
	40	80	200	400	1000	1024	2000	4000
L2	0.347	0.341	0.094	0.098	0.019	0.026	0.012	0.008
L3	0.333	0.333	0.333	0.333	0.333	0.208	0.333	0.333
L4	0.506	0.436	0.507	0.420	0.555	0.605	0.514	0.511
L6	0.433	0.381	0.355	0.314	0.351	0.578	0.351	0.344
L7	0.543	0.471	0.111	0.109	0.015	0.068	0.009	0.004
Arbitrary	0.192	0.207	0.132	0.140	0.134	0.138	0.130	0.131
Matching	0.347	0.333	0.333	0.333	0.333	0.333	0.333	0.333
Paths	0.562	0.567	0.350	0.375	0.192	—	0.171	0.143
Regions	0.196	0.220	0.133	0.135	0.133	0.135	0.134	0.131
Scheduling	0.317	0.249	0.190	0.252	0.202	0.114	0.195	0.216
Size	1000/500		1000/1000		1500/1500			
LG	0.317		0.249		0.190			

B Statistical tests

Tables 2–9, show U test results for each pair of algorithms and different instance sizes, at 99% of confidence level. The structure of these tables is as follows: Each row and column is indexed by one algorithm. Each element in the diagonal (bold) is the median of the corresponding algorithm. The upper-right diagonal elements are the differences in location statistics for each pair of algorithms. A positive difference indicates that the “row algorithm” has its location statistics higher (better) than the “column algorithm”, and the negative difference is the opposite. The bottom-left diagonal elements are the p -values of each test. We omitted all $p < 0.01$ values, that indicate that the difference is statistically significant for those pairs. We also omitted confidence intervals since for all tests the values lie in these intervals and they are very narrow. For instance, in Table 2 we can see that the location statistics for CPLEX (2nd line) are higher (better) than for RG_{RK} (4th column) since the value 0.0806 is positive. Since the p -value for this pair was omitted (3rd line, 3rd column), the table indicates that CPLEX performed significantly better than RG_{RK} in these tests. We chose to display a large number of significant digits since for some pairs of algorithms the differences are very small they are still statistically significant. This is the case, for example, of algorithms GA_{RA} and SD_{LP} in Table 2 where the difference is only 0.000009 but is still significant (in terms of the U test) in favor of GA_{RA}.

Since several tests were performed, we applied a p -value correction procedure based on false discovery rate (Benjamini and Hochberg, 1995) aiming to minimize the number of false positives (Type I error).

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Table 2: Difference in median location for cost distributions for all instances, using a confidence interval of 99%. The omitted p -values are less than 0.0009.

	CORAL	CPLEX	RG _{RK}	BO _{MA}	CA _{RA}	CA _{LP}	GA _{RA}	GA _{LP}	SD _{RA}	SD _{LP}
CORAL	0.000000	-0.950720	-0.840038	-0.916819	-0.999908	-0.999944	-0.988866	-0.999941	-0.930584	-0.961331
CPLEX		0.982822	0.080600	0.018096	-0.001477	-0.003086	-0.000017	-0.002008	0.004974	-0.000078
RG _{RK}			0.875503	-0.051401	-0.114096	-0.115350	-0.104717	-0.114523	-0.062547	-0.088295
BO _{MA}				0.939643	-0.051826	-0.053154	-0.042700	-0.051845	-0.000850	-0.025455
CA _{RA}					1.000000	-0.000036	0.000002	0.000014	0.034406	0.000695
CA _{LP}						1.000000	0.000005	0.000037	0.035707	0.003093
GA _{RA}							1.000000	-0.000067	0.026002	0.000012
GA _{LP}					$p > 0.29$			1.000000	0.034429	0.001266
SD _{RA}									0.955629	-0.010476
SD _{LP}		$p > 0.07$								0.951900

Table 3: Difference in median location for cost distributions for instances with 400 bids or less, using a confidence interval of 99%. The omitted p -values are less than 0.004.

	CORAL	CPLEX	RG _{RK}	BO _{MA}	CA _{RA}	CA _{LP}	GA _{RA}	GA _{LP}	SD _{RA}	SD _{LP}
CORAL	0.999984	-0.000062	-0.000052	-0.000033	-0.000012	-0.000042	-0.000031	-0.000037	-0.000017	-0.000039
CPLEX		1.000000	0.000083	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
RG _{RK}			1.000000	-0.000049	-0.000057	-0.000029	-0.000070	-0.000077	-0.000007	-0.000026
BO _{MA}				1.000000	-0.000039	-0.000011	-0.000041	-0.000016	-0.000046	-0.000025
CA _{RA}					1.000000	-0.000047	0.000012	-0.000077	0.000063	-0.000068
CA _{LP}		$p > 0.09$				1.000000	0.000071	0.000022	0.000060	-0.000089
GA _{RA}					$p > 0.25$		1.000000	-0.000037	0.000041	-0.000007
GA _{LP}		$p > 0.08$				$p > 0.86$		1.000000	0.000083	-0.000032
SD _{RA}							$p > 0.01$		1.000000	-0.000076
SD _{LP}		$p > 0.30$								0.999300

Table 4: Difference in median location for cost distributions for instances with 1000 bids or more, using a confidence interval of 99%. The omitted p -values are less than 0.001.

	CORAL	CPLEX	RG _{RK}	BO _{MA}	CA _{RA}	CA _{LP}	GA _{RA}	GA _{LP}	SD _{RA}	SD _{LP}
CORAL	0.000000	-0.946779	-0.856397	-0.922194	-0.999946	-0.999927	-0.990230	-0.999915	-0.931543	-0.957204
CPLEX		0.953821	0.079737	0.022089	-0.035372	-0.035907	-0.023595	-0.034921	0.014744	-0.000004
RG _{RK}			0.863907	-0.056439	-0.124063	-0.124426	-0.112816	-0.123114	-0.062318	-0.088282
BO _{MA}				0.926657	-0.062474	-0.062700	-0.053631	-0.061591	-0.005172	-0.029241
CA _{RA}					1.000000	-0.000065	0.000065	0.000018	0.051891	0.027844
CA _{LP}						1.000000	0.000086	0.000071	0.052948	0.028406
GA _{RA}							0.993222	-0.000044	0.043039	0.017885
GA _{LP}					$p > 0.27$			1.000000	0.051161	0.027183
SD _{RA}									0.938537	-0.021078
SD _{LP}		$p > 0.03$								0.951700

Table 5: Difference in median location for cost distributions for LG 1500/1500 instances, using a confidence interval of 99%. The omitted p -values are less than 0.00001.

	RG _{RK}	BO _{MA}	CA _{RA}	CA _{LP}	GA _{RA}	GA _{LP}	SD _{RA}	SD _{LP}		
RG _{RK}	0.827570	-0.081040	-0.163875	-0.162074	-0.154572	-0.160352	-0.102957	-0.109679		
BO _{MA}		0.910548	-0.083920	-0.083313	-0.076227	-0.079144	-0.022304	-0.028658		
CA _{RA}			1.000000	0.000027	0.000036	0.000045	0.057928	0.053322		
CA _{LP}				$p > 0.05$	1.000000	0.000036	0.000057	0.057150	0.052600	
GA _{RA}						0.998888	-0.000042	0.050617	0.042824	
GA _{LP}				$p > 0.01$			1.000000	0.054981	0.047942	
SD _{RA}								0.935822	-0.005051	
SD _{LP}									$p > 0.01$	0.942900

Table 6: Difference in median location of cost distributions for all instances, considering the best solutions until 100 generations. A confidence interval of 99% was used. The omitted p -values are less than 0.000009.

	RG _{RK}	BO _{MA}	CA _{RA}	CA _{LP}	GA _{RA}	GA _{LP}	SD _{RA}	SD _{LP}	
RG _{RK}	0.365256	-0.027354	-0.484062	-0.495047	-0.439106	-0.495536	-0.082448	-0.251942	
BO _{MA}		0.410741	-0.433579	-0.443894	-0.389594	-0.444571	-0.028155	-0.208201	
CA _{RA}			0.971370	-0.000033	0.000045	-0.000016	0.313978	0.115553	
CA _{LP}				0.995975	0.000040	0.000046	0.321063	0.124509	
GA _{RA}					0.894743	-0.000053	0.274147	0.069280	
GA _{LP}				$p > 0.84$			0.993376	0.321429	0.126418
SD _{RA}								0.541106	-0.126061
SD _{LP}									0.721500

Table 7: Difference in median location of cost distributions for instances with 400 bids or less, considering the best solutions until 100 generations. A confidence interval of 99% was used. The omitted p -values are less than 0.009.

	RG _{RK}	BO _{MA}	CA _{RA}	CA _{LP}	GA _{RA}	GA _{LP}	SD _{RA}	SD _{LP}
RG _{RK}	0.713545	-0.000017	-0.000011	-0.000029	-0.000019	-0.000020	-0.000009	-0.000034
BO _{MA}		1.000000	0.000005	0.000051	0.000030	0.000067	0.000038	0.000059
CA _{RA}	$p > 0.01$		0.990978	-0.000027	0.000008	-0.000006	-0.000000	-0.000015
CA _{LP}			$p > 0.01$	0.999988	0.000003	0.000033	0.000021	-0.000044
GA _{RA}			$p > 0.37$	$p > 0.11$	0.999943	-0.000037	0.000018	-0.000040
GA _{LP}			$p > 0.04$	$p > 0.70$	$p > 0.27$	0.999986	0.000039	-0.000049
SD _{RA}	$p > 0.41$		$p > 0.37$		$p > 0.09$	$p > 0.01$	0.793841	-0.000020
SD _{LP}				$p > 0.52$	$p > 0.03$	$p > 0.37$		1.000000

Table 8: Difference in median location of cost distributions for instances with more than 400 bids, considering the best solutions until 100 generations. A confidence interval of 99% was used. The omitted p -values are less than 0.0001.

	RG _{RK}	BO _{MA}	CA _{RA}	CA _{LP}	GA _{RA}	GA _{LP}	SD _{RA}	SD _{LP}
RG _{RK}	0.342195	-0.020136	-0.548485	-0.557676	-0.502515	-0.556347	-0.132994	-0.300077
BO _{MA}		0.370348	-0.521230	-0.531241	-0.477689	-0.530029	-0.115826	-0.284344
CA _{RA}			0.968853	-0.000039	0.009928	-0.000032	0.365621	0.198026
CA _{LP}				0.987577	0.021097	0.000033	0.375408	0.208959
GA _{RA}					0.889071	-0.020640	0.324260	0.154933
GA _{LP}				$p > 0.70$		0.988826	0.375060	0.208850
SD _{RA}							0.530006	-0.153962
SD _{LP}								0.694700

Table 9: Difference in median location of cost distributions for for LG 1500/1500 instances, considering the best solutions until 100 generations. A confidence interval of 99% was used. The omitted p -values are less than 0.00004.

	RG _{RK}	BO _{MA}	CA _{RA}	CA _{LP}	GA _{RA}	GA _{LP}	SD _{RA}	SD _{LP}
RG _{RK}	0.228827	-0.064896	-0.661234	-0.661403	-0.608903	-0.665038	-0.242486	-0.306383
BO _{MA}		0.312164	-0.599983	-0.601567	-0.544650	-0.604113	-0.172053	-0.239376
CA _{RA}			0.991615	-0.000039	0.009253	-0.000065	0.418796	0.334902
CA _{LP}			$p > 0.97$	0.991615	0.012762	-0.000056	0.419685	0.334964
GA _{RA}					0.890265	-0.018571	0.351878	0.290690
GA _{LP}			$p > 0.68$	$p > 0.68$		0.993368	0.421907	0.336256
SD _{RA}							0.479321	-0.057048
SD _{LP}								0.548400

C Additional running time results

Table 10 shows the average time in seconds taken by each algorithm to find the best solution (recall that we limited runs to at most 3,600 seconds). The additional time in the last iterations without improvement in the best solution found is disregarded. We also exclude the time used loading instances and logging. To be fair with the Java implementations, each run began with a warm-up phase so that Java virtual machine could load and optimize all necessary bytecode. The first two columns of this table list, respectively, the instance classes and their corresponding sizes. Each following pair of columns shows the average time and standard deviation for each algorithm, respectively.

Table 10: Running time comparison among the algorithms. For each algorithm, it is shown the average time to find the best solutions. The over time used in the last iterations without improvement is disregard. Time in seconds.

Class	Size	CORAL		CPLEX		RG _{RK}		BO _{MA}		CA _{RA}		CA _{LP}		GA _{RA}		GA _{LP}		SD _{RA}		SD _{LP}	
		Time	σ	Time	σ	Time	σ	Time	σ	Time	σ	Time	σ	Time	σ	Time	σ	Time	σ	Time	σ
CATS	40	1	1	1	1	10	6	1	1	1	0	1	0	2	1	1	0	2	3	1	0
	80	2	2	1	1	12	7	1	1	2	1	1	0	5	38	1	0	2	4	1	0
	200	792	1438	1	1	18	15	17	32	14	74	1	0	31	123	1	0	46	132	1	0
	400	923	1418	1	1	40	56	51	121	58	161	28	124	47	139	18	96	65	172	20	89
	1000	2883	1436	382	1075	473	651	736	936	53	136	30	96	60	134	36	104	75	167	32	112
	1024	2803	1494	960	1568	460	591	778	1130	54	142	33	114	72	165	30	106	79	181	28	96
	2000	2903	1411	1377	1708	1463	1151	1909	1324	72	181	35	106	66	135	39	114	78	166	29	99
	4000	3012	1344	1802	1799	2527	1540	2611	1361	52	115	28	75	68	133	42	119	54	123	30	87
LG	1000	3606	12	3601	1	289	529	75	68	94	196	93	192	94	196	95	197	78	181	71	168
	1500	3624	23	3601	1	425	702	66	58	118	211	113	200	94	187	100	187	92	188	83	179

D Results for parameter tuning**RGRK**

```

# Best candidates
popsize tournamentsize pertubation
  434             18      0.1493
  472             16      0.1488
  437             16      0.1459

```

BOMA

```

# Best candidates
popsize highqualityindividuas diversifiedindividuals maxlocalsearchiters
  1377             12             24             142
   516             16             26             238
  1178             11             24             115

```

BRKGAs

```

# Best candidates
  pe      pm      rhoe indpop intervalexchange elitexchange
0.2116 0.1639 0.7609     3           136           2
0.2527 0.0679 0.7698     3           127           1
0.2297 0.0741 0.7976     3           141           1

```

E Best results for each instance

This section presents the results obtained for LG instances. The tables format is the following: the first column and second columns are the instance name and the best revenue obtained for this instance, respectively. The following columns show the percentage of the revenue from the best solution obtained by the algorithm that names the column. A high percentage indicates that the obtained solution is closer to the best. A star (*) indicates that the algorithm found the best solution.

Table 11: Best results for CATS instances with less than 400 bids. The names of the instances are composed by the class, number of bids, number of goods, and serial number of the instance.

Inst.	Best	CORAL	CPLEX	RG _{RR}	BO _{MA}	CA _{RA}	CA _{LP}	GA _{RA}	GALP	SD _{RA}	SD _{LP}
L2.40.10.1	8774.7200	*	*	*	*	*	*	*	*	*	*
L2.40.10.2	9229.3400	*	*	*	*	*	*	*	*	*	*
L2.40.10.3	8967.4300	*	*	*	*	*	*	*	*	*	*
L2.80.10.1	9828.2500	77.64	*	*	*	*	*	*	*	*	*
L2.80.10.2	9786.7100	*	*	*	*	*	*	*	*	*	*
L2.80.10.3	9441.1700	*	*	*	*	*	*	*	*	*	*
L2.200.50.2	45785.7000	*	*	*	*	*	*	*	*	92.15	*
L2.200.50.3	49031.9000	87.41	*	*	*	*	*	*	*	*	*
L2.400.50.1	46588.7410	*	*	*	*	*	*	*	*	98.34	*
L2.400.50.2	47706.0000	*	*	*	*	*	*	*	*	*	*
L2.400.50.3	47819.7160	*	*	*	*	*	*	*	*	*	*
L3.40.10.1	2474.2480	*	*	*	*	*	*	*	*	*	*
L3.40.10.2	2682.7890	*	*	*	*	*	*	*	*	*	*
L3.40.10.3	2929.2870	*	*	*	*	*	*	*	*	*	*
L3.80.10.1	2862.1650	*	*	*	*	*	*	*	*	*	*
L3.80.10.2	2779.9100	*	*	*	*	*	*	*	*	*	*
L3.80.10.3	2938.3120	*	*	*	*	*	*	*	*	*	*
L3.200.50.1	12178.8010	*	*	*	*	*	*	*	*	*	*
L3.200.50.3	12612.9650	*	*	*	*	*	*	*	*	*	*
L3.400.50.1	14338.1150	*	*	*	*	*	*	*	*	*	*
L3.400.50.2	14747.9490	*	*	99.07	*	*	*	*	*	*	*
L3.400.50.3	14495.9880	*	*	99.54	*	*	*	*	*	*	*
L4.40.10.1	9543.8540	*	*	*	*	*	*	*	*	*	*
L4.40.10.2	8870.7760	*	*	*	*	*	*	*	*	*	*
L4.40.10.3	9249.9330	*	*	*	*	*	*	*	*	*	*
L4.80.10.1	9770.0770	*	*	99.69	*	*	*	*	*	*	*
L4.80.10.2	9817.6040	*	*	99.50	*	*	*	*	*	*	*
L4.80.10.3	9759.7910	*	*	*	*	*	*	*	*	*	*
L4.200.50.1	45191.2690	86.61	*	99.65	*	*	*	*	*	*	*
L4.200.50.2	44275.5990	92.24	*	99.56	*	*	*	*	*	*	*
L4.200.50.3	46496.4650	93.73	*	*	*	*	*	*	*	*	*
L4.400.50.1	47748.4440	89.23	*	99.42	*	*	*	*	*	*	*
L4.400.50.2	47988.4200	*	*	99.56	99.62	*	*	*	*	*	*
L4.400.50.3	48410.5140	*	*	99.40	99.55	*	*	*	*	*	*
L6.40.10.1	8791.5910	*	*	*	*	*	*	*	*	*	*
L6.40.10.2	9297.1700	*	*	*	*	*	*	*	*	*	*
L6.40.10.3	9217.2400	*	*	*	*	*	*	*	*	*	*
L6.80.10.1	9290.9270	*	*	*	*	*	*	*	*	*	*
L6.80.10.2	9836.4500	*	*	*	*	*	*	*	*	*	*
L6.80.10.3	9593.9010	*	*	97.69	*	*	*	*	*	*	*
L6.200.50.1	41639.9910	96.86	*	95.31	*	*	*	*	*	*	*
L6.200.50.2	38873.5410	98.44	*	99.62	*	*	*	*	*	99.53	*
L6.200.50.3	40561.3300	*	*	*	*	*	*	*	*	*	*
L6.400.50.1	44990.9010	99.12	*	*	*	*	*	*	*	*	*
L6.400.50.2	46366.8710	99.45	*	97.48	*	*	*	*	*	*	*
L6.400.50.3	45216.8660	96.18	*	96.83	95.86	99.92	*	*	*	*	*
L7.40.10.1	8309.1230	*	*	*	*	*	*	*	*	*	*
L7.40.10.2	9090.6580	*	*	*	*	*	*	*	*	*	*
L7.40.10.3	8553.2690	*	*	*	*	*	*	*	*	*	*
L7.80.10.1	9818.5880	*	*	99.32	*	*	*	*	*	*	*
L7.80.10.2	9435.4580	*	*	*	*	*	*	*	*	*	*
L7.80.10.3	9775.6220	*	*	99.81	*	*	*	*	*	*	*
L7.200.50.1	28286.0100	*	*	*	*	*	*	*	*	*	*

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Table 11: (continued).

Inst.	Best	CORAL	CPLEX	RG _{RR}	BO _{MA}	CA _{RA}	CA _{LP}	GA _{RA}	GA _{LP}	SD _{RA}	SD _{LP}
L7_200_50.2	30478.8250	69.46	*	*	*	*	*	*	*	*	*
L7_200_50.3	29014.3000	*	*	*	*	*	*	*	*	*	*
L7_400_50.1	32505.1200	*	*	*	*	*	*	*	*	98.15	*
L7_400_50.2	33512.5500	*	*	96.52	*	*	*	*	*	*	*
L7_400_50.3	29829.2200	*	*	98.74	*	*	*	*	*	*	*
arbitrary_40_10.1	1019.7600	*	*	*	*	*	*	*	*	*	*
arbitrary_40_10.2	749.5520	99.98	*	*	*	*	*	*	*	*	*
arbitrary_40_10.3	679.8568	*	*	*	*	*	*	*	*	*	*
arbitrary_80_10.1	1038.7469	*	*	*	*	*	*	*	*	*	*
arbitrary_80_10.2	775.6040	*	*	*	*	*	*	*	*	*	*
arbitrary_80_10.3	581.9593	*	*	*	*	*	*	*	*	*	*
arbitrary_200_50.1	3007.5090	*	*	96.00	*	*	*	*	*	99.13	*
arbitrary_200_50.2	3260.1100	*	*	97.54	*	*	*	*	*	*	*
arbitrary_200_50.3	3271.8422	*	*	96.60	*	*	*	*	*	*	*
arbitrary_400_50.1	4038.0004	*	*	93.09	90.48	*	*	*	*	*	*
arbitrary_400_50.2	3791.8860	*	*	93.80	85.02	*	*	*	*	*	*
arbitrary_400_50.3	4289.3898	*	*	88.70	*	*	*	*	*	*	*
matching_40_10.1	10.7792	*	*	*	*	*	*	*	*	*	*
matching_40_10.2	7.1517	*	*	*	*	*	*	*	*	*	*
matching_40_10.3	15.6388	*	*	*	*	*	*	*	*	*	*
matching_80_10.1	13.7825	*	*	*	*	*	*	*	*	*	*
matching_80_10.2	1.5328	*	*	*	*	*	*	*	*	*	*
matching_80_10.3	8.1853	*	*	*	*	*	*	*	*	*	*
matching_200_50.1	32.2974	*	*	*	*	*	*	*	*	*	*
matching_200_50.2	32.0509	*	*	*	*	*	*	*	*	*	*
matching_200_50.3	23.8792	*	*	*	*	*	*	*	*	*	*
matching_400_50.1	41.8873	*	*	*	*	*	*	*	*	*	*
matching_400_50.2	55.8732	*	*	*	*	*	*	*	*	*	*
matching_400_50.3	27.1398	*	*	*	*	*	*	*	*	*	*
paths_40_10.1	4.5826	*	*	*	*	*	*	*	*	*	*
paths_40_10.2	6.1875	*	*	*	*	*	*	*	*	*	*
paths_40_10.3	5.3516	*	*	*	*	*	*	*	*	*	*
paths_80_10.1	7.0575	*	*	*	*	*	*	*	*	*	*
paths_80_10.2	5.0659	*	*	*	*	*	*	*	*	*	*
paths_80_10.3	6.0272	*	*	*	*	*	*	*	*	*	*
paths_200_50.1	20.2063	97.32	*	*	*	*	*	*	*	*	*
paths_200_50.2	20.0969	*	*	*	*	*	*	*	*	*	*
paths_200_50.3	22.1016	*	*	*	*	*	*	*	*	*	*
paths_400_50.1	26.8886	*	*	*	99.54	*	*	*	*	*	*
paths_400_50.2	22.9762	*	*	*	*	*	*	*	*	*	*
paths_400_50.3	23.7947	*	*	*	99.72	*	*	*	*	*	*
regions_40_10.1	942.2510	*	*	*	*	*	*	*	*	*	*
regions_40_10.2	750.5340	*	*	*	*	*	*	*	*	*	*
regions_40_10.3	659.7730	*	*	*	*	*	*	*	*	*	*
regions_80_10.1	808.6960	*	*	96.58	*	*	*	*	*	*	*
regions_80_10.2	957.2460	*	*	*	*	*	*	*	*	*	*
regions_80_10.3	1159.6219	*	*	98.13	98.13	*	*	*	*	*	*
regions_200_50.1	3616.3098	*	*	97.69	*	*	*	*	*	*	*
regions_200_50.2	3292.0154	*	*	87.48	*	*	*	*	*	*	*
regions_200_50.3	3401.2610	*	*	92.77	*	*	*	*	*	*	*
regions_400_50.1	4177.5069	*	*	89.33	*	*	*	*	*	*	*
regions_400_50.2	3606.1991	*	*	91.53	*	*	*	*	*	*	*
regions_400_50.3	3482.6069	*	*	93.81	*	*	*	*	*	*	*
scheduling_40_10.1	14.7840	*	*	*	*	*	*	*	*	*	*
scheduling_40_10.2	15.1235	*	*	*	*	*	*	*	*	*	*
scheduling_40_10.3	21.7354	*	*	*	*	*	*	*	*	*	*
scheduling_80_10.1	22.6557	*	*	*	*	*	*	*	*	*	*
scheduling_80_10.2	15.7918	*	*	*	*	*	*	*	*	*	*
scheduling_80_10.3	22.8672	*	*	*	*	*	*	*	*	*	*
scheduling_200_50.1	22.6139	*	*	*	*	*	*	*	*	*	*
scheduling_200_50.2	53.8402	*	*	*	*	*	*	*	*	*	*
scheduling_200_50.3	48.8045	*	*	95.26	*	*	*	*	*	*	*
scheduling_400_50.1	58.2749	*	*	97.38	*	*	*	*	*	*	*
scheduling_400_50.2	70.3185	*	*	*	*	*	*	*	*	*	*
scheduling_400_50.3	43.8167	1.50	*	*	*	*	*	*	*	*	*

Table 12: Best results for CATS instances more than 400 bids. The names of the instances are composed by the class, number of bids, number of goods, and serial number of the instance. Instances with `hard` in the name have 1024 bids and 256 goods.

Inst.	Best	CORAL	CPLEX	RG _{RK}	BO _{MA}	C _{RA}	C _{LP}	G _{RA}	G _{LP}	SD _{RA}	SD _{LP}
L2.1000_256.1	244098.0000	*	*	*	*	*	*	*	*	88.40	*
L2.1000_256.2	241158.0000	*	*	*	*	*	*	*	*	90.88	*
L2.1000_256.3	254988.0000	*	*	*	*	*	*	*	*	81.73	*
L2.2000_512.1	495448.0000	*	*	*	*	*	*	*	*	72.84	*
L2.2000_512.2	501810.0000	4.27	*	*	*	*	*	*	*	80.74	*
L2.2000_512.3	505625.0000	*	*	*	*	*	*	*	*	87.47	*
L2.4000_1024.1	1000590.0000	*	*	*	*	*	*	*	*	37.53	*
L2.4000_1024.2	1010991.6920	10.66	*	*	100.00	*	*	*	*	59.42	*
L2.4000_1024.3	996744.0000	*	*	*	*	*	*	*	*	34.50	*
L2.hard.1	262.5110	*	*	*	*	*	*	*	*	63.91	*
L2.hard.2	456.5370	*	*	*	*	*	*	*	*	*	*
L2.hard.3	317.4120	*	*	*	*	*	*	*	*	66.52	*
L3.1000_256.1	64626.2530	75.64	*	99.26	99.60	99.60	99.60	99.60	99.56	96.60	99.50
L3.1000_256.2	66106.1710	79.20	*	99.08	99.31	99.75	99.78	99.78	99.66	98.58	99.78
L3.1000_256.3	64987.7430	75.45	*	99.68	*	99.84	*	99.81	*	99.79	*
L3.2000_512.1	128004.7893	81.84	*	98.80	97.74	99.47	99.92	99.92	99.72	98.43	99.95
L3.2000_512.2	132229.2010	93.59	*	99.18	98.78	99.49	99.71	99.35	99.78	97.62	99.69
L3.2000_512.3	133133.1410	82.48	*	98.95	98.83	99.07	99.64	99.43	99.62	97.09	99.62
L3.4000_1024.1	263970.8210	78.25	*	90.99	97.45	99.88	99.74	98.87	99.50	96.46	99.25
L3.4000_1024.2	263936.8590	75.72	*	91.47	97.29	99.48	99.62	99.60	99.55	96.62	99.53
L3.4000_1024.3	263404.1096	80.03	*	91.46	96.98	99.09	99.30	99.17	98.94	96.70	98.93
L3.hard.1	75.4074	75.32	*	99.35	97.11	99.28	99.39	99.18	99.35	98.89	99.18
L3.hard.2	34.1897	80.59	99.33	96.96	95.45	99.27	*	99.09	97.74	97.17	97.74
L3.hard.3	20.8636	82.47	96.79	97.63	96.19	98.44	98.63	98.44	*	95.31	*
L4.1000_256.1	228752.1550	4.41	*	99.52	99.17	99.85	*	*	*	99.67	*
L4.1000_256.2	229601.7270	92.34	*	99.27	98.48	99.71	*	*	*	99.43	*
L4.1000_256.3	229349.1950	2.85	*	99.58	99.70	99.90	*	*	*	99.43	*
L4.2000_512.1	461218.2640	3.56	*	99.48	99.04	99.71	*	*	*	99.07	*
L4.2000_512.2	459425.3230	2.75	*	99.83	98.97	99.73	*	*	*	99.32	*
L4.2000_512.3	458536.7260	2.64	*	99.50	99.07	99.65	*	*	*	99.50	*
L4.4000_1024.1	914322.9910	2.30	*	98.33	96.11	99.32	*	*	*	99.15	*
L4.4000_1024.2	920786.4330	2.22	*	98.39	95.88	99.09	*	*	*	98.93	*
L4.4000_1024.3	920294.1540	3.27	*	98.37	96.36	99.16	99.99	99.99	99.99	99.00	99.99
L4.hard.1	290.2399	16.81	*	99.53	98.80	*	*	*	*	*	*
L4.hard.2	383.8526	13.76	*	99.36	99.14	*	*	*	*	*	*
L4.hard.3	282.6879	7.06	*	99.55	98.90	*	*	*	*	*	*
L6.1000_256.1	199757.0790	45.81	*	97.22	99.41	98.23	98.75	98.32	98.72	98.22	98.22
L6.1000_256.2	200559.8373	69.95	*	97.41	96.57	98.80	99.39	97.61	99.39	97.38	99.11
L6.1000_256.3	201208.1706	3.72	*	99.56	99.15	98.72	99.96	98.36	98.92	97.16	98.36
L6.2000_512.1	405788.3937	72.95	*	98.00	95.24	97.76	99.00	97.85	99.00	94.39	99.00
L6.2000_512.2	411091.1370	5.22	*	97.83	94.81	97.77	98.16	97.22	98.16	96.02	97.77
L6.2000_512.3	402472.3077	71.43	*	98.29	95.41	97.04	97.93	97.73	97.93	94.55	97.93
L6.4000_1024.1	785686.0929	0.88	*	98.15	93.36	97.30	97.62	97.54	97.73	97.60	97.75
L6.4000_1024.2	801026.0135	75.21	*	98.90	92.95	97.91	97.66	97.33	97.84	95.53	95.95
L6.4000_1024.3	791849.8150	73.60	*	98.19	93.02	97.06	97.03	96.92	97.38	95.83	96.82
L6.hard.1	377.5873	13.69	*	99.57	99.51	*	*	*	*	*	*
L6.hard.2	330.2240	18.25	*	99.58	99.65	*	*	*	*	*	*
L6.hard.3	446.4472	6.50	*	99.62	99.68	*	*	*	*	*	*
L7.1000_256.1	68830.4000	95.17	*	*	*	*	*	*	*	86.30	*
L7.1000_256.2	79025.8000	96.74	*	*	100.00	*	*	*	*	96.74	*
L7.1000_256.3	81981.6000	100.00	*	*	100.00	*	*	*	*	*	*
L7.2000_512.1	121043.0000	*	*	*	*	*	*	*	*	97.56	*
L7.2000_512.2	119058.0000	*	*	*	*	*	*	*	*	93.95	*
L7.2000_512.3	122346.0000	99.99	*	*	*	*	*	*	*	92.35	*
L7.4000_1024.1	244374.0000	*	*	*	*	*	*	*	*	90.59	*
L7.4000_1024.2	229826.0000	*	*	*	*	*	*	*	*	99.68	*
L7.4000_1024.3	228342.0000	*	*	*	*	*	*	*	*	88.19	*
L7.hard.1	233.0348	73.22	*	*	*	*	*	98.25	*	*	*
L7.hard.2	127.4510	100.00	*	*	*	*	*	*	*	*	*
L7.hard.3	261.2782	83.15	97.72	97.70	*	99.18	*	99.18	*	95.56	95.56
arbitrary.1000_256.1	17186.3016	70.40	96.39	93.12	95.91	*	*	96.87	*	94.46	95.53
arbitrary.1000_256.2	15782.8217	6.06	98.02	96.27	95.63	98.41	*	98.56	99.40	96.39	98.03
arbitrary.1000_256.3	17280.1375	9.31	98.04	92.55	97.45	98.69	*	99.28	99.28	97.28	97.28
arbitrary.2000_512.1	32267.8600	0.17	96.98	96.15	93.59	99.74	98.89	*	99.56	95.35	96.13
arbitrary.2000_512.2	32159.7621	1.56	95.83	96.42	94.28	99.97	*	99.39	99.19	98.02	97.21

Continue in next page...

Table 12: (continued).

Inst.	Best	CORAL	CPLEX	RG _{RK}	BO _{MA}	CA _{RA}	CA _{LP}	GA _{RA}	GA _{LP}	SD _{RA}	SD _{LP}
arbitrary_2000.512.3	32181.8011	2.52	95.86	96.79	97.81	99.54	99.27	*	99.04	97.77	98.95
arbitrary_4000.1024.1	62694.3745	1.43	95.70	93.86	91.57	99.64	98.48	98.56	*	96.90	96.76
arbitrary_4000.1024.2	61809.4598	0.35	97.64	93.42	90.90	97.84	98.90	*	97.92	96.32	95.92
arbitrary_4000.1024.3	62366.9031	79.65	96.20	93.46	90.80	98.82	99.01	97.84	*	96.39	95.77
arbitrary_hard.1	16412.4678	1.32	99.78	94.33	95.55	*	99.78	98.07	99.78	95.12	96.07
arbitrary_hard.2	15699.7262	71.07	98.47	96.52	98.13	*	98.47	98.02	98.47	98.26	99.65
arbitrary_hard.3	14954.8919	1.29	99.40	95.34	98.76	99.91	99.43	99.69	99.43	*	*
matching_1000.256.1	724.3030	26.06	*	99.88	99.88	*	*	*	*	*	*
matching_1000.256.2	731.8279	94.91	*	99.92	99.99	*	*	*	*	99.99	*
matching_1000.256.3	912.8670	92.44	*	*	99.97	*	*	*	*	99.82	*
matching_2000.512.1	669.1193	30.47	*	99.91	99.51	*	*	*	*	99.84	*
matching_2000.512.2	1379.1604	92.92	*	99.72	99.48	*	*	*	*	99.96	*
matching_2000.512.3	881.3102	24.14	*	99.93	99.35	*	*	*	*	99.76	*
matching_4000.1024.1	3047.5592	64.56	*	94.35	98.41	99.98	*	*	*	99.82	*
matching_4000.1024.2	2302.0147	38.07	*	94.29	98.19	100.00	*	*	*	99.78	*
matching_4000.1024.3	2508.6253	31.33	*	95.11	98.39	99.94	*	*	*	99.87	*
matching_hard.1	155.0591	*	*	*	*	*	*	*	*	*	*
matching_hard.2	421.5402	23.49	*	99.96	99.96	*	*	*	*	*	*
matching_hard.3	323.9873	94.06	*	*	99.99	*	*	*	*	*	*
paths_1000.256.1	57.7328	89.93	*	*	*	*	*	*	*	*	*
paths_1000.256.2	65.7292	28.17	*	*	98.37	*	*	*	*	*	*
paths_1000.256.3	57.4862	30.51	*	*	*	*	*	*	*	99.17	*
paths_2000.512.1	90.3558	62.85	*	100.00	95.94	*	*	*	*	*	*
paths_2000.512.2	101.4873	52.98	*	99.83	97.01	*	*	100.00	99.98	99.21	99.98
paths_2000.512.3	106.9681	57.30	*	99.80	97.50	99.92	99.87	99.87	99.87	99.41	99.87
paths_4000.1024.1	161.5959	99.87	*	98.26	91.88	99.70	*	*	*	98.50	*
paths_4000.1024.2	165.5882	80.80	*	98.52	93.20	99.47	*	*	*	98.79	*
paths_4000.1024.3	150.9125	97.21	*	98.70	91.61	99.82	*	*	*	98.99	*
regions_1000.256.1	16214.3571	74.32	*	95.20	98.99	98.94	99.36	98.12	99.36	98.09	98.89
regions_1000.256.2	17922.5058	75.94	*	95.60	99.63	99.23	98.67	99.10	98.79	98.87	98.67
regions_1000.256.3	17391.3627	3.78	*	97.09	*	99.54	99.54	98.18	99.34	97.85	99.34
regions_2000.512.1	38262.6408	74.08	*	97.36	97.88	98.30	99.55	98.68	98.99	97.65	98.86
regions_2000.512.2	32274.1576	62.09	*	95.82	95.49	99.77	98.64	97.89	98.60	96.23	97.45
regions_2000.512.3	37199.7468	1.20	*	96.77	98.55	99.21	99.59	98.49	99.50	97.50	98.95
regions_4000.1024.1	65807.6502	0.33	99.97	94.38	96.23	99.56	99.28	97.14	*	96.09	96.67
regions_4000.1024.2	65628.9304	2.72	99.70	95.57	97.09	99.91	*	98.25	99.30	98.75	98.97
regions_4000.1024.3	64800.3099	5.43	*	95.15	95.79	99.06	99.04	96.26	99.42	94.39	95.07
regions_hard.1	15336.4868	72.73	*	94.01	98.24	99.82	99.82	99.22	99.82	98.55	99.82
regions_hard.2	17988.3370	70.19	*	94.17	99.48	99.48	99.74	98.55	99.74	99.74	99.48
regions_hard.3	16777.2344	72.67	*	93.30	98.03	99.17	99.66	98.75	99.66	97.94	98.35
scheduling_1000.256.1	44.9038	22.99	*	*	*	*	*	*	*	*	*
scheduling_1000.256.2	42.5548	20.98	*	*	*	*	*	*	*	*	*
scheduling_1000.256.3	87.6889	11.41	*	*	*	*	*	*	*	*	*
scheduling_2000.512.1	40.9792	25.74	*	*	*	*	*	*	*	*	*
scheduling_2000.512.2	58.2106	3.63	*	*	*	*	*	*	*	*	*
scheduling_2000.512.3	48.2352	1.32	*	*	*	*	*	*	*	*	*
scheduling_4000.1024.1	28.3994	*	*	*	*	*	*	*	*	*	*
scheduling_4000.1024.2	45.9743	*	*	*	*	*	*	*	*	*	*
scheduling_4000.1024.3	36.6752	*	*	*	*	*	*	*	*	*	*
scheduling_hard.1	168.4070	98.37	*	*	*	*	*	*	*	*	*
scheduling_hard.2	1219.4075	100.00	*	*	*	*	*	*	*	*	*
scheduling_hard.3	4812.6430	56.44	*	*	*	*	*	*	*	*	*

Table 13: Best results for LG 1000/500 instances.

Inst.	Best	CORAL	CPLEX	RG _{RK}	BO _{MA}	CA _{RA}	CA _{LP}	GA _{RA}	GA _{LP}	SD _{RA}	SD _{LP}
in101	72724.6180	37.66	92.27	92.27	96.03	*	*	*	*	95.02	98.63
in102	72518.2220	45.17	98.03	96.72	98.22	*	*	*	*	97.44	99.82
in103	72129.5000	40.67	96.64	95.13	96.76	*	*	97.41	*	*	98.43
in104	72709.6470	65.30	98.04	94.46	97.42	*	*	*	*	92.45	*
in105	75646.1406	39.96	89.05	90.91	*	*	*	*	*	94.99	*
in106	71258.6130	51.23	89.77	93.23	94.31	*	*	*	*	94.49	*
in107	69713.4030	38.64	98.38	98.38	99.24	*	*	99.55	*	*	*
in108	75813.2109	11.33	98.39	99.12	*	99.95	*	99.12	99.95	99.31	99.30
in109	69475.8950	38.47	91.99	95.09	95.34	*	*	*	*	*	*

Continue in next page...

Table 13: (continued).

Inst.	Best	CORAL	CPLEX	RG _{RRK}	BO _{MA}	CA _{RA}	CA _{LP}	GA _{RA}	GA _{LP}	SD _{RA}	SD _{LP}
in110	68295.2890	16.29	*	92.75	99.79	*	*	*	*	*	*
in111	75133.2900	42.87	96.74	95.16	97.12	*	*	*	*	95.53	97.12
in112	71342.4830	60.02	99.25	94.80	99.81	*	*	*	*	*	*
in113	73365.8906	53.84	92.73	96.02	*	*	*	*	*	98.43	*
in114	69224.7656	13.31	94.35	94.58	*	*	98.58	99.56	99.56	*	96.04
in115	70221.5610	48.33	94.85	93.15	96.06	*	*	99.55	*	95.95	96.95
in116	70032.4609	48.56	98.32	93.80	*	*	*	*	*	98.32	98.32
in117	69982.8330	59.34	94.96	99.92	99.92	*	*	*	*	95.33	98.99
in118	72160.9870	57.56	95.02	93.06	97.21	*	*	*	*	95.36	95.36
in119	67038.4297	58.44	96.86	*	*	*	*	98.36	*	98.27	98.27
in120	75514.9300	58.27	98.85	93.41	99.95	*	*	99.13	99.13	97.67	98.87
in121	67639.1250	34.44	96.47	94.24	*	*	*	*	*	96.34	96.34
in122	69546.2730	40.98	96.73	98.24	98.24	*	*	99.97	*	95.69	*
in123	70618.3130	49.50	92.25	94.96	99.97	*	*	*	99.97	98.78	99.93
in124	71686.0469	39.54	97.28	99.72	*	*	*	*	*	96.63	99.72
in125	69233.1220	51.98	95.79	95.79	97.41	*	*	*	*	98.14	*
in126	70671.7700	9.53	98.45	93.05	98.61	*	*	98.61	*	95.98	96.62
in127	69273.3203	42.94	98.39	92.27	*	*	*	*	*	98.74	*
in128	72179.4310	17.30	94.35	90.70	98.32	*	*	*	*	95.27	96.20
in129	65751.6490	37.24	97.51	97.51	97.59	*	*	*	*	97.51	97.51
in130	71075.3000	48.78	97.39	97.90	97.90	*	*	99.14	*	96.04	97.90
in131	71177.9062	2.62	95.49	99.62	*	*	*	*	*	96.39	*
in132	75510.0469	43.34	96.88	99.90	*	*	*	*	*	*	*
in133	71253.5610	54.48	97.85	94.16	99.35	*	*	99.35	*	94.95	97.67
in134	75781.7490	46.70	96.61	91.22	98.73	*	*	*	*	97.39	*
in135	72138.1172	2.42	95.49	90.72	*	*	*	*	*	*	*
in136	68903.0938	43.37	94.79	96.29	*	*	*	99.86	99.86	96.61	99.04
in137	70072.0469	48.96	*	90.56	*	*	*	99.99	*	*	*
in138	71989.6330	28.25	97.43	99.24	99.24	99.24	*	99.24	*	99.24	97.71
in139	72840.3940	35.02	94.24	92.53	98.79	*	*	*	*	96.13	98.94
in140	73665.2310	43.72	*	92.15	92.42	*	*	*	*	*	*
in141	69605.0770	40.43	98.91	96.15	99.67	*	*	*	*	95.42	98.91
in142	74777.9850	49.90	97.26	94.59	96.20	*	*	*	*	97.52	97.52
in143	69699.0547	34.12	95.14	98.81	*	*	*	*	*	98.19	98.89
in144	73197.0730	49.56	94.95	93.48	99.03	*	*	*	*	*	*
in145	73695.0150	39.38	96.88	92.77	96.25	*	*	97.81	*	*	97.80
in146	73746.9375	38.30	95.29	93.65	*	*	*	*	*	97.29	97.29
in147	65878.3020	58.53	95.28	97.17	97.17	*	*	*	*	94.88	94.88
in148	72116.9690	51.94	96.01	95.66	98.84	99.81	*	98.84	*	99.81	99.81
in149	70800.1800	46.53	97.27	95.68	98.61	*	*	99.30	*	99.02	*
in150	72839.4240	46.46	94.35	93.20	98.91	*	*	*	*	*	*
in151	68834.5010	45.83	99.99	97.90	99.13	*	*	*	*	98.85	99.75
in152	76224.7812	41.04	93.71	93.62	*	*	*	*	*	97.94	97.94
in153	70110.7650	43.46	99.49	96.81	99.49	*	*	99.60	*	96.54	98.00
in154	69215.5240	7.16	94.14	96.66	98.48	*	99.27	99.27	99.27	99.09	99.27
in155	74936.7730	36.64	96.51	96.26	97.22	*	*	99.75	*	99.75	*
in156	69704.1300	50.74	93.01	99.24	99.24	*	*	*	*	96.64	96.64
in157	73934.8438	33.75	91.20	93.38	*	*	*	*	*	92.71	*
in158	69489.5430	47.97	*	92.69	97.71	*	*	*	*	*	95.13
in159	71091.8047	55.58	96.38	95.46	*	*	*	*	*	98.53	98.53
in160	70606.9180	46.45	96.31	99.48	99.48	*	*	*	*	98.61	98.61
in161	66266.3710	15.81	92.56	93.91	98.53	*	99.34	*	*	97.88	*
in162	74720.7940	54.27	93.32	95.58	97.44	99.44	99.44	*	*	99.44	99.44
in163	64976.9910	46.23	98.63	98.45	99.06	*	99.86	99.86	99.86	99.06	99.06
in164	67950.6230	43.67	93.99	91.64	99.41	*	*	*	*	98.38	98.38
in165	70361.9531	39.37	95.13	95.97	*	*	*	98.61	*	97.19	97.19
in166	71460.8930	34.20	92.35	95.05	97.99	99.80	*	99.45	99.80	97.56	97.83
in167	74523.7656	26.61	*	96.58	*	*	*	*	*	96.86	96.86
in168	72097.3210	43.31	97.37	96.68	96.86	*	*	99.54	*	*	*
in169	71827.3400	45.18	96.43	97.32	98.45	*	*	98.62	*	95.21	*
in170	74564.7490	42.64	92.70	92.51	95.80	*	*	96.46	*	90.03	95.15
in171	71279.4840	37.03	96.53	94.32	97.33	*	*	98.48	*	98.48	98.45
in172	70361.8070	3.68	99.57	93.91	96.82	*	*	99.57	*	97.41	98.66
in173	73677.2030	57.65	96.29	93.20	99.78	*	*	*	*	94.10	97.49
in174	73523.6094	44.59	96.31	92.89	*	*	*	*	*	96.48	95.46
in175	72924.8740	49.45	97.54	91.49	97.26	*	99.67	99.67	99.67	99.67	99.67
in176	67761.4830	38.10	97.51	95.42	99.35	*	*	*	*	98.33	99.43
in177	70187.1540	49.13	94.27	95.49	98.94	*	*	*	*	98.88	98.94

Continue in next page...

Table 13: (continued).

Inst.	Best	CORAL	CPLEX	RG _{RK}	BO _{MA}	CA _{RA}	CA _{LP}	GA _{RA}	GA _{LP}	SD _{RA}	SD _{LP}
in178	70833.3720	53.70	90.85	92.84	95.71	*	*	*	*	94.42	95.20
in179	72205.2980	42.51	96.80	95.57	96.60	*	*	98.97	*	96.60	*
in180	70513.3520	54.50	93.16	94.07	96.53	*	*	*	*	96.26	96.31
in181	72238.0859	37.33	95.06	97.16	*	*	*	*	*	96.76	99.07
in182	71645.0312	37.70	97.82	*	*	*	*	*	*	98.80	98.80
in183	71520.4688	37.89	98.38	93.86	*	*	*	*	*	99.57	99.57
in184	74377.5380	1.74	92.64	87.92	94.41	*	*	*	*	*	*
in185	73714.9531	47.24	*	94.44	*	*	*	99.52	*	99.52	*
in186	70736.2480	47.66	97.98	94.98	98.72	*	*	*	*	97.98	97.98
in187	70166.3660	31.20	95.26	94.28	97.34	*	*	*	*	98.55	98.55
in188	70485.1950	40.11	93.17	95.47	96.52	*	*	98.86	*	99.49	*
in189	69786.0220	38.77	95.35	96.84	98.82	*	*	*	*	98.54	*
in190	73765.2090	38.54	97.07	97.07	98.60	*	*	*	*	*	99.72
in191	72587.0780	8.24	99.65	97.87	98.63	*	*	99.65	*	*	*
in192	71156.8280	34.93	93.02	94.22	99.45	*	*	*	*	99.61	99.61
in193	72526.4688	33.95	97.21	94.22	*	*	*	*	*	97.46	97.46
in194	75803.5156	47.87	94.14	*	*	*	*	*	*	99.91	99.91
in195	69066.8672	29.99	91.41	96.21	*	*	*	*	*	96.25	96.25
in196	69776.2220	51.81	98.39	98.47	99.91	*	*	99.91	*	98.70	97.77
in197	68457.8040	55.49	*	*	98.33	*	*	*	*	97.41	97.41
in198	73474.3830	41.26	98.84	92.37	97.19	*	*	*	*	97.19	97.19
in199	70955.9130	37.03	93.84	95.59	99.98	*	*	99.98	*	98.21	98.34
in200	76803.1830	46.02	95.19	95.17	98.09	*	*	98.88	*	*	*

Table 14: Best results for LG 1000/1000 instances.

Inst.	Best	CORAL	CPLEX	RG _{RK}	BO _{MA}	CA _{RA}	CA _{LP}	GA _{RA}	GA _{LP}	SD _{RA}	SD _{LP}
in201	81557.7578	45.28	94.10	*	*	*	*	*	*	99.71	96.02
in202	90708.1406	38.91	98.60	93.30	*	*	*	*	*	99.96	*
in203	86239.2266	7.67	95.45	93.96	*	*	*	99.12	*	97.69	97.69
in204	87075.4453	38.39	94.14	94.39	*	*	*	*	*	95.49	98.62
in205	86515.9510	34.40	93.59	92.44	97.11	*	*	*	*	94.14	96.80
in206	91518.9640	19.12	94.93	93.41	94.93	*	*	*	*	*	*
in207	93129.2900	27.22	*	97.75	99.99	*	*	99.99	*	94.91	94.91
in208	94904.6953	25.73	88.80	90.18	*	*	*	96.71	*	96.71	96.71
in209	87268.9650	47.58	98.88	93.37	99.41	*	*	99.41	*	98.88	96.83
in210	89962.4062	39.71	96.64	95.73	*	*	*	*	*	98.38	*
in211	84913.6840	55.07	93.20	92.87	99.54	*	*	*	*	97.60	98.48
in212	90778.2188	40.06	96.81	91.38	*	*	*	*	*	98.97	98.97
in213	85369.1850	34.71	95.81	97.87	97.87	*	*	*	*	98.97	98.97
in214	85181.6090	39.16	97.34	96.19	99.58	*	*	*	*	99.58	*
in215	91531.7031	46.31	95.42	93.46	*	*	*	*	*	99.56	97.91
in216	91580.9800	48.61	*	93.53	94.72	*	*	*	*	*	*
in217	86962.9270	52.45	97.72	93.77	98.33	*	*	*	*	97.72	99.92
in218	94965.2109	45.14	90.34	91.55	*	*	*	*	*	*	*
in219	93586.4380	46.99	90.94	96.02	96.02	*	*	*	*	96.08	96.08
in220	89792.9219	44.44	97.87	96.82	*	*	*	98.48	*	98.63	98.63
in221	87410.7800	41.62	*	93.56	97.23	*	*	*	*	96.00	96.23
in222	89905.5391	45.82	94.77	90.80	*	*	*	*	*	*	*
in223	83045.4297	40.13	96.04	88.95	*	*	*	*	*	94.30	94.71
in224	87105.2770	49.10	96.86	98.39	99.92	*	*	*	*	97.13	97.40
in225	89430.1094	38.68	95.90	91.21	*	*	*	*	*	*	*
in226	88176.1220	34.96	91.75	90.75	95.92	*	*	95.09	*	95.92	95.92
in227	92613.3710	44.80	96.95	95.57	98.94	*	*	*	*	*	*
in228	92684.0781	56.28	96.70	96.70	*	*	*	*	*	98.86	95.33
in229	90468.1420	49.34	96.50	91.38	96.75	*	*	*	*	96.76	96.76
in230	91559.1562	48.44	96.66	94.13	*	*	*	*	*	97.74	97.74
in231	101458.6094	40.11	93.07	88.09	*	*	*	*	*	*	*
in232	87270.8630	17.55	95.18	91.66	99.45	*	*	*	*	92.66	*
in233	86151.8980	39.85	96.84	94.09	98.81	*	*	*	*	96.91	97.09
in234	88874.3359	49.60	98.17	92.70	*	*	*	*	*	96.84	96.84
in235	93246.5700	38.90	*	89.98	*	*	*	*	*	93.01	97.24
in236	87876.7891	38.94	98.10	91.59	*	*	*	*	*	95.25	96.84
in237	87616.0450	54.09	96.48	94.61	98.30	*	*	*	*	97.16	99.78
in238	87004.0781	49.72	98.22	90.70	*	*	*	99.57	*	98.70	*

Continue in next page...

Table 14: (continued).

Inst.	Best	CORAL	CPLEX	RG _{RK}	BO _{MA}	CA _{RA}	CA _{LP}	GA _{RA}	GA _{LP}	SD _{RA}	SD _{LP}
in239	81435.3020	41.92	99.86	92.88	99.86	*	*	*	*	98.41	98.41
in240	86608.4120	45.38	98.11	98.61	98.61	*	*	*	*	95.04	98.90
in241	89961.1641	39.00	98.80	92.63	*	*	*	*	*	98.80	98.80
in242	92480.5420	35.73	90.80	91.44	92.68	*	*	*	*	95.12	96.56
in243	91839.5970	37.24	99.91	91.99	99.91	*	*	*	*	96.47	96.47
in244	91029.7940	42.40	95.61	92.89	98.11	*	*	98.11	*	97.04	97.04
in245	90590.5630	34.27	95.38	96.10	96.10	*	*	*	*	94.46	99.06
in246	87158.2344	24.83	99.39	*	*	*	*	99.17	*	99.39	99.17
in247	89044.3828	45.42	96.32	96.01	*	*	*	*	*	99.54	99.54
in248	93058.1406	57.39	91.53	92.92	*	*	*	*	*	95.73	95.73
in249	95169.5190	62.17	93.98	93.98	98.01	*	*	*	*	96.22	96.22
in250	93775.8359	48.37	*	*	*	*	*	*	*	98.82	98.82
in251	88734.0770	43.94	92.56	92.35	96.09	*	*	96.42	*	96.11	96.11
in252	89504.9220	53.90	93.49	98.03	98.03	*	*	*	*	99.86	99.86
in253	88253.3125	24.40	95.78	96.70	*	*	*	*	*	96.87	*
in254	85897.5010	31.00	96.01	96.37	96.37	*	*	*	*	98.85	*
in255	89368.1990	37.11	94.69	94.32	98.13	*	*	97.74	*	98.67	98.67
in256	89253.2656	38.86	93.03	92.12	*	*	*	96.74	*	*	95.03
in257	88605.5950	12.67	96.54	94.88	99.49	*	*	*	*	97.23	99.17
in258	85183.9110	44.59	99.33	97.74	98.65	*	*	*	*	*	*
in259	95397.3516	37.58	*	87.77	*	*	*	*	*	93.80	93.80
in260	90407.2050	42.48	99.25	92.46	96.03	*	*	*	*	99.38	*
in261	89790.1900	46.72	*	92.80	*	*	*	*	*	97.30	97.30
in262	88470.1100	50.02	*	92.98	99.03	*	*	*	*	96.68	*
in263	93087.8530	37.55	94.59	94.24	97.35	*	*	*	*	98.98	98.98
in264	86498.9141	48.56	97.86	91.86	*	*	*	*	*	99.00	99.00
in265	83621.1700	41.51	95.48	97.91	98.94	*	*	98.47	*	*	99.16
in266	90038.9920	31.15	96.12	94.84	98.48	*	*	*	*	98.50	98.50
in267	91438.2109	24.48	99.40	92.66	*	*	*	*	*	*	*
in268	89482.2790	41.41	97.93	93.04	99.78	*	*	*	*	98.80	98.80
in269	83546.6830	48.56	99.19	96.77	99.88	*	*	*	*	99.46	99.46
in270	87509.4062	34.87	97.20	92.81	*	*	*	*	*	95.73	95.73
in271	85951.6810	42.83	95.87	93.42	97.72	*	*	*	*	98.51	98.51
in272	88642.8220	49.07	92.82	95.86	97.28	*	*	*	*	*	*
in273	87909.9070	39.27	99.20	95.06	99.96	*	*	99.21	*	*	*
in274	83417.7890	45.77	98.89	93.02	99.18	*	*	98.89	*	98.33	98.33
in275	89915.1500	37.12	98.05	94.19	99.17	*	*	99.57	*	98.79	98.79
in276	86626.4375	50.65	99.36	99.40	*	*	*	99.78	*	97.18	99.36
in277	88537.7270	37.49	96.56	98.09	98.79	*	*	98.79	*	96.58	96.58
in278	91326.9531	52.88	95.55	96.72	*	*	*	99.28	*	99.28	99.28
in279	87058.9800	46.83	*	89.91	96.27	*	*	98.88	*	97.82	98.45
in280	86529.5938	38.92	97.14	93.11	*	*	*	*	*	99.46	99.46
in281	88470.4141	53.98	96.51	95.42	*	*	*	*	*	99.75	99.75
in282	88985.3290	46.25	92.23	91.56	96.27	*	*	*	*	95.57	96.27
in283	88915.6590	49.94	95.47	96.33	96.33	*	*	*	*	*	*
in284	88241.9750	39.20	96.30	96.86	96.86	*	*	*	*	*	97.14
in285	85953.2490	45.59	96.57	93.35	97.89	*	*	*	*	99.16	99.16
in286	88323.4844	57.41	92.98	*	*	*	*	*	*	92.53	92.31
in287	91652.7400	32.42	99.46	88.37	99.46	*	*	*	*	93.58	*
in288	85639.0090	41.68	95.93	96.81	97.90	*	*	*	*	96.81	96.17
in289	86032.8140	49.01	96.03	91.94	96.03	*	*	*	*	97.12	97.48
in290	92103.2070	42.87	95.79	90.63	96.06	*	*	*	*	94.55	95.24
in291	94188.2910	59.11	92.98	95.55	96.30	*	*	*	*	94.64	94.64
in292	94063.9650	57.29	97.14	95.96	96.56	*	*	*	*	97.90	*
in293	85810.6210	51.78	98.18	95.25	98.82	*	*	*	*	97.04	*
in294	91167.3160	49.30	94.62	91.92	96.80	*	*	*	*	*	*
in295	89267.5156	34.74	94.01	93.08	*	*	*	*	*	*	95.56
in296	90000.2970	22.43	98.55	93.35	98.55	*	*	*	*	*	*
in297	89725.9360	27.45	*	94.05	94.84	*	*	*	*	97.43	97.43
in298	89166.7422	47.96	98.65	93.52	*	*	*	*	*	*	98.77
in299	92218.6094	41.60	98.03	95.49	*	*	*	99.53	*	93.10	93.10
in300	88373.3281	48.68	97.91	*	*	*	*	*	*	99.30	96.75

Table 15: Best results for LG 1500/1500 instances.

Inst.	Best	CORAL	CPLEX	RG _{RK}	BO _{MA}	CA _{RA}	CA _{LP}	GA _{RA}	GA _{LP}	SD _{RA}	SD _{LP}
in601	108800.4450	58.25	95.76	91.03	96.77	*	*	98.18	*	97.43	97.43
in602	105611.4760	24.41	93.92	92.95	95.78	*	*	*	*	94.12	94.12
in603	105121.0220	39.04	92.40	88.06	92.54	*	*	97.57	*	*	*
in604	107733.8050	50.52	96.29	96.13	96.29	98.96	98.96	98.96	*	98.04	98.04
in605	109840.9840	52.38	93.23	92.38	94.98	*	*	*	*	*	*
in606	107113.0670	37.26	92.47	97.42	98.23	*	*	*	*	93.83	93.83
in607	113180.2840	43.74	90.23	93.54	93.54	*	*	*	*	91.09	*
in608	105266.1070	50.50	96.26	88.48	97.88	*	*	*	*	99.07	99.07
in609	109472.3320	3.23	96.71	90.87	95.77	*	*	*	*	94.18	95.55
in610	113716.9650	32.91	93.89	88.04	95.86	*	*	98.04	*	95.87	98.04
in611	106666.3438	10.31	94.57	88.69	*	*	*	*	*	98.76	98.76
in612	109796.7400	54.31	*	96.59	*	*	*	*	*	96.91	94.91
in613	107980.1570	71.82	93.49	86.40	93.09	*	*	*	*	96.58	*
in614	108364.5859	49.66	*	89.98	*	*	*	*	*	*	*
in615	110508.8281	37.36	97.15	86.14	*	*	*	98.92	*	92.40	92.40
in616	109740.4922	44.02	88.30	91.67	*	*	*	*	*	95.39	96.64
in617	113302.4340	45.99	92.27	91.02	93.78	*	*	*	*	91.75	95.29
in618	111385.0810	47.03	94.81	88.56	99.93	*	*	99.58	*	95.53	99.57
in619	107571.5930	43.20	97.72	90.46	94.97	*	*	*	*	97.72	*
in620	110937.9750	59.54	92.13	93.65	95.07	*	*	97.96	*	97.96	96.75
in621	106133.8500	41.62	*	93.40	93.40	*	*	*	*	98.14	98.14
in622	107551.7370	55.58	91.12	94.14	98.49	*	*	*	*	96.71	97.53
in623	109487.0290	42.38	94.77	94.57	94.99	*	*	*	*	96.90	96.90
in624	104386.9790	48.61	92.76	92.27	95.69	*	*	*	*	*	*
in625	109065.3594	43.83	96.90	88.55	*	*	*	*	*	94.30	97.57
in626	114704.0340	50.12	89.36	88.28	96.60	*	*	*	*	97.77	92.88
in627	108846.2344	37.55	91.65	95.31	*	*	*	99.17	*	99.17	99.17
in628	108169.6953	42.91	94.53	*	*	*	*	97.07	*	97.51	96.74
in629	107929.2600	40.10	95.76	94.64	97.16	*	*	*	*	98.12	98.29
in630	105830.0620	54.00	94.55	93.68	99.65	*	*	*	*	99.75	99.75
in631	116505.2440	31.18	94.57	94.02	96.91	*	*	*	*	*	*
in632	104631.7140	52.88	92.98	90.76	95.18	*	*	99.59	*	98.28	*
in633	105564.4000	65.72	*	90.90	98.72	*	*	*	*	94.20	94.02
in634	108901.7300	47.86	94.31	91.85	93.34	*	*	*	*	93.80	93.80
in635	112902.6340	39.75	92.44	86.62	92.44	*	*	*	*	94.72	94.72
in636	106574.7480	48.82	92.64	91.03	98.23	*	*	99.07	*	97.76	93.89
in637	107989.7280	33.07	92.70	91.77	99.01	*	*	*	*	99.01	99.01
in638	112899.6320	30.01	97.48	88.95	92.88	*	*	97.48	*	97.48	97.48
in639	108894.4550	43.77	92.99	94.68	95.35	*	*	*	*	*	*
in640	108275.1328	53.59	96.04	91.48	*	*	*	99.08	*	96.60	96.60
in641	109744.0625	56.06	98.77	92.27	*	*	*	99.73	*	98.77	98.77
in642	114182.9688	40.41	91.12	90.13	*	*	*	*	*	*	*
in643	104015.0240	13.04	94.62	91.75	97.97	*	*	*	*	97.97	*
in644	108025.7490	60.10	98.22	93.57	98.34	*	*	99.03	*	99.03	98.25
in645	105841.6720	37.99	92.97	90.05	96.51	*	*	*	*	96.08	97.25
in646	107800.1030	33.71	93.84	94.90	95.98	*	*	*	*	*	*
in647	107701.7109	51.25	90.90	93.95	*	*	*	95.92	*	95.37	95.37
in648	105790.5900	37.28	*	96.66	*	*	*	99.93	*	99.55	99.55
in649	107587.3710	40.30	88.79	94.52	95.70	*	*	98.63	*	*	99.79
in650	103330.9010	45.80	92.36	93.79	96.86	*	*	*	*	97.02	97.50
in651	103827.2970	55.97	95.17	94.21	98.85	*	*	*	*	*	*
in652	107760.2480	28.48	94.24	97.48	97.48	*	*	*	*	97.20	96.19
in653	113946.4766	34.60	91.38	89.41	*	*	*	*	*	94.22	94.22
in654	111738.2310	35.91	94.25	89.29	98.26	*	*	*	*	*	*
in655	111785.0640	44.33	91.74	88.65	97.39	*	*	*	*	96.13	96.13
in656	112259.2750	43.93	90.25	96.67	96.67	*	*	*	*	94.21	95.51
in657	112708.6560	37.47	*	93.86	96.88	*	*	*	*	95.65	*
in658	110751.5340	38.17	91.70	91.29	93.87	*	*	96.80	*	*	*
in659	106545.4270	39.45	94.76	96.16	96.16	*	*	99.03	*	*	*
in660	112293.6080	39.96	98.65	91.81	96.61	*	*	99.72	*	99.72	98.65
in661	113106.6290	30.29	97.20	87.06	92.81	*	*	*	*	95.86	97.63
in662	108298.0790	58.18	97.41	91.26	91.26	*	*	*	*	94.29	94.29
in663	104826.7800	52.39	95.93	95.40	95.40	*	*	*	*	99.03	92.33
in664	112866.8650	42.89	95.93	91.67	94.66	*	*	99.38	*	99.68	99.68
in665	113002.6720	39.05	98.78	94.75	97.33	*	*	98.78	*	96.68	96.68
in666	106441.1562	46.49	*	91.88	*	*	*	*	*	98.54	98.54
in667	104683.7500	65.93	97.55	91.77	*	*	*	*	*	97.75	97.75
in668	107483.1580	45.33	94.12	93.41	98.89	*	*	*	*	98.12	98.12

Continue in next page...

Table 15: (continued).

Inst.	Best	CORAL	CPLEX	RG _{RR}	BO _{MA}	CA _{RA}	CA _{LP}	GA _{RA}	GA _{LP}	SD _{RA}	SD _{LP}
in669	108163.4690	42.49	97.80	93.65	96.78	*	*	*	*	95.71	94.23
in670	110200.8160	50.35	94.90	92.73	96.53	*	*	*	*	99.98	99.85
in671	109306.8438	48.13	99.75	*	*	*	*	*	*	99.75	99.75
in672	107534.8870	43.05	93.33	95.58	95.58	*	*	*	*	96.40	95.58
in673	112320.2500	44.61	92.20	92.34	*	*	*	*	*	98.43	98.43
in674	109558.2344	37.01	91.87	87.59	*	*	*	*	*	95.40	95.40
in675	108131.9880	47.04	*	92.14	97.81	*	*	*	*	*	*
in676	107052.1910	37.62	94.64	88.05	96.10	*	*	*	*	*	*
in677	107831.5370	45.33	96.19	97.47	99.68	*	*	99.68	*	97.03	97.22
in678	102422.8290	29.45	*	95.83	96.87	*	*	*	*	*	*
in679	107982.4560	48.90	90.90	92.06	99.21	*	*	98.61	*	96.35	96.11
in680	107500.5000	44.67	96.69	91.50	*	*	*	*	*	98.92	98.92
in681	105237.2870	53.94	93.19	92.10	99.85	*	*	*	*	*	*
in682	107948.1260	38.39	97.33	89.67	98.25	*	*	*	*	98.25	99.93
in683	107777.6130	7.06	95.19	93.47	96.08	*	*	*	*	98.25	98.25
in684	114153.7410	62.07	91.14	85.74	94.52	*	*	*	*	92.55	94.26
in685	106686.6160	39.69	94.83	92.42	92.74	*	*	97.71	*	97.81	97.81
in686	106364.3580	19.52	99.45	92.48	98.55	*	99.53	99.45	99.53	97.70	97.70
in687	108301.4710	44.81	97.06	94.98	97.05	*	*	*	*	99.10	99.10
in688	112012.5703	50.12	93.49	94.55	*	*	*	99.83	*	95.27	97.60
in689	105968.1680	48.45	92.72	94.96	97.70	*	*	*	*	98.39	98.39
in690	108489.7109	34.23	92.05	92.90	*	*	*	*	*	97.02	97.02
in691	105564.6090	37.21	93.06	96.78	*	*	*	*	*	98.50	98.50
in692	109226.0700	44.39	93.71	91.40	97.15	*	*	*	*	98.99	97.40
in693	106719.6950	31.31	97.08	93.34	97.58	*	*	99.56	*	99.56	97.08
in694	114477.0540	47.90	89.45	94.44	94.44	*	*	96.94	*	93.66	93.66
in695	110240.9860	14.09	91.30	93.91	93.91	*	*	98.04	*	*	*
in696	104559.9530	39.47	*	95.00	98.94	*	*	*	*	99.43	99.43
in697	105958.6570	23.47	98.78	92.49	98.46	*	*	*	*	98.78	98.78
in698	105463.0312	26.21	95.12	92.14	*	*	*	*	*	97.86	97.77
in699	107132.3340	41.24	96.37	96.85	98.42	*	*	99.26	*	99.14	99.14
in700	106730.6770	45.46	95.37	95.11	95.11	*	*	*	*	97.09	94.31

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